Double Meandering Algorithm: 
From Drawing Game to Automated Animation

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Abstract
We introduce artist Lucy Pullen’s Double Meandering Algorithm, first in its original form as a pen-and-paper drawing algorithm and then as a procedurally generated animation. We utilize a chain of cubic Bézier curves to represent the characteristic spiraling line, assigning each control point according to a pseudo-randomized algorithm. The resulting curves are then animated segment by segment, reflecting the artist’s process of creating the pen-and-paper drawing. By digitizing the Double Meandering Line drawing, we can also reveal the process of creation through animation, granting us the ability to exhibit a fundamental part of the drawing that is lost in the traditional pen-and-paper presentation.

Categories and Subject Descriptors (according to ACM CCS): J.5 [Computer Graphics]: Computer Applications—Fine Arts

1. Introduction

We present a digital representation of a pen-and-paper ‘drawing game’ called the Double Meandering Line. The Double Meandering Line is the novel creation of Lucy Pullen, professor and Conceptual artist based in New York and Victoria, B.C.

The Double Meandering Line is a form of generative art based on a set of predetermined rules; a finished drawing is simply an outcome of following simple two-dimensional rules for laying down each iteration of curves. The resulting shapes, however, appear to be three-dimensional after completion. A key feature of the drawing is the process, not just the context of the finished work. To experience the contrast, the viewer must witness the drawing in the making – a luxury which is not afforded by the traditional gallery model. Thus, we endeavor to create a procedurally generated animation that bypasses the inherent limitations of both static ‘finished’ art and process art.

Our digital version, the Double Meandering Algorithm, attempts to capture the three-dimensional shape achieved by the artist by combining the same sort of segment drawing rules with chains of cubic Bézier curves and pseudo-randomized variables to generate a digital drawing with similar aesthetics.

The boundary between science and the arts is alternately judged to be either imaginary or impermeable, depending on the matter at hand – imaginary when the project is conceptualized, and impermeable when the objectives are not being reached as planned. Interdisciplinary projects are historically fraught with tension due to an unexpected inability for peo-
ple with such different backgrounds to reconcile their differences. Ultimately, the success of an interdisciplinary project comes down to the ability and willingness of both parties to communicate. In this paper we also report of discoveries made as computer scientists try to blend their skills with the talents of an artist.

2. Related Work

Modernists break the rules. Minimalists make rules of their own to break. Our shared interest as researchers in art and computer science is in new imagery. To make new images, we have taken our own process and are attempting to recreate it in another domain – we are creating rules for the explicit purpose of breaking them. This job exceeds the scope of a single creative individual and discipline. We are working with inside information on a particular artistic method, in the domain of computer science.

The artist is actively involved in the planning and development of the procedurally generated animation described in this paper, making the system a collaborative work rather than a derivative on the part of a research group. The analog Double Meandering Line already exists as a pre-existing process, and the primary goal is to emulate the outward appearance of that process. Thus, our work on the system is cumulative of the outcomes of that process, and should be considered in the context of other style-emulative work.

2.1. Background in Visual Art

Guidelines for technical drawings of industrial objects were first described by the French Academy in the late 1800’s. In her essay ‘The Language of Industry’, Molly Nesbitt discusses how Marcel Duchamp used these rules to create new modes of abstraction [Nes86]. Common objects such as the shovel and the urinal appeared as art object Readymades as early as 1917, thus breaking the classifications established by the French Academy.

It is accepted practice in both Minimalism and Conceptual art for the artist to be removed from the physical construction of the finished piece. Keystone Minimalist Donald Judd had sculptures constructed by factory contractors, and Lawrence Weiner produced pieces that consisted of short instructions to be carried out by gallery staff onsite [?]. In both cases, the only name attached to the creative aspect of work is that of the artist, and we adhere to this conventionhere.

Generative art is associated with practices where the artist cedes control of the outcome to a self-supporting system, such as a set of rules or a computer [Gal03]. Though the term itself is a 20th century invention, the principles have been in use for far longer – the definition of a self-supporting system could be as broad as to include the use of concepts as universal as symmetry. The movement encompasses not only visual art, but also music and literature. It is one of the principal movements to deal with the element of randomness, which can often be the main driving force of the work.

As computers are one of the primary platforms for self-supporting systems at our disposal today, a good portion of computer-assisted art falls under the generative art heading. Additionally, web application platform such as Flash and Processing have given generative artists a convenient medium in which to develop and package their work. For example, Jared Tarbell maintains a large collection of his algorithmic art on the internet, complete with source code [Tar10].

2.2. Background in Computer Graphics

The fields of Computational Aesthetics and Non-Photorealistic Rendering originate in the emulation of the personal styles of visual artists. For example, work by Lee et. al introduces a fluid jet simulation that allows a user to produce pictures in the style of Jackson Pollock [LOG06]. Xu et. al. developed a system that replicates traditional Chinese ink paintings as digital strokes, which can be manipulated to create animations based on the original paintings [?].

This work as a programming artifact has intents in common with generative art and computational aesthetics [?], but our Double Meandering Algorithm is heavily based on the traditional drawing rules of one specific set of works by Lucy Pullen.

3. Double Meandering Line Drawing

A firm understanding of the Double Meandering Line drawing is a prerequisite to the development of the digital algorithm. Though differences between the analog and digital processes are inevitable, we want to adhere as close as possible to the original process. The Double Meandering Line drawing is not generative art, but the algorithm is generative by necessity. We have concentrated on the generative art principles of the Double Meandering Line drawing in our analysis, as it is the most effective paradigm for our purposes.

Complexity in the Double Meandering Line drawing is the accumulation of many simple parts. The drawing consists of two parallel lines following a curve, from the beginning point of each spiral to the end without touching. Together they create the impression of breadth. Each curve comes back to near where they started and once the lines intersect, they stop and then both lines continue on the other side, one leads, the other follows, as if forming a knot or length of rope. However, this is not a drawing of a knot. This is a simple set of two rules, creating a deliberate manufacture of coherent abstraction.

In this paper we present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. We present a study and implementation based on the Double Meandering Line drawing. 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Figure 2: Progression of a Double Meandering Line drawing spiral from start to finish, by Lucy Pullen.

3.1. Terminology
First, we will describe some terminology that will be used throughout this paper. A spiral can be described as a series of curlicues with connected ends. We describe a single curlicue in this configuration as an iteration. This step-by-step approach to the spiral structure is invaluable to understanding and emulating the Double Meandering Line drawing.

The curlicues of the spiral define the mutual path of two parallel lines. The line drawn on the inside edge of the spine is always the first one to be drawn, so we will refer to it as the original line and the outside line the following line.

Through parameterization and randomization we control the spine or offset path of the curlicues into the page, generating the appearance of the 3D shape found in the Double Meandering Line drawing.

3.2. Step by Step Analysis
As a "drawing game", the Double Meandering Line (DML) drawing requires no tools save for a drawing surface and a pen. Though more complex, post-spiral DML drawings can span large swaths of paper, the technique is scaleable to any size. Generally, two to four of variously sized and shaped spirals are drawn on the same page, overlapping each other. The combination of multiple spirals furthers the sense of depth and adds complexity to the composition.

The process of creating a DML drawing of the spiral type is a straightforward, step-by-step process. The Double Meandering Line has two stages: an Opening stage and a Closing stage. The Opening Stage starts with a self-intersecting curlicue, with the following line running parallel to the first and ending upon contact with the original line. The next curlicue is a larger version of the first iteration, with the original and following lines starting close to where they left off, but not quite – they start from a point of contact with the following line rather than exactly where they left off, illustrated in Figure 2. This creates an illusion of overlap and depth as if the double lines are a single entity, similar to the 'rope' found in Celtic knots.

In the drawing, this pattern continues until an apparent bias towards a particular side of the iteration has been formed, at which point the iterations enter the Closing stage and become exaggerated into a tapered tail end. The tail end becomes thin and elongated to the point where it cannot contain the next iteration. Then, the line stops iterating and trails off in the direction of the tail. The result is a conch-like shape that appears to be made of thick wire.

Double Meandering Line drawings depend on the interrelation between an iteration and its predecessors. The shape of each iteration can be predicted by the previous iteration, but cannot be fully explained; the irregularities of the predecessor are amplified with each iteration.

4. Procedurally Generated Animation
From the analysis of the Double Meandering Line drawing, we gather a set of requirements for a program that will emulate the drawings with reasonable accuracy. The computer algorithm’s success lies in the accurate emulation of an ac-
ceptable spiral shape, balancing randomization with authenticity.

We require a structure that easily accommodates a smoothly spiraling line. The structure should be aware enough of the shapes of the previous iterations to imitate the overall shape, while also having the flexibility to change drastically between iterations.

For our initial algorithm we considered two-dimensional spirals, such as the logarithmic spiral, since spirals are already similar to the shape of the Double Meandering Line drawings and can be described using a single polar coordinate equation. However, they are not sufficiently malleable—one cannot independently warp the shape of a particular iteration without affecting the shapes of the preceding or succeeding iterations. The shape of the spiral iterations can only be modified via a global matrix warp, which of course cannot warp a single iteration of the line without affecting the others.

Instead, we take advantage of the flexibility and controlability of drawing with splines. The use of splines is inefficient compared to the use of spiral primitives, as even a single Bézier curve is composed of multiple equations. A chain of Bézier curves also requires storage for each and every node position in the chain. In this case, accuracy of shape takes precedent over speed, which makes the splines a natural choice. A chain of Bézier curves, linked from end to end, creates a malleable curved line that can serve as our spine. Though the spiral arrangement must be created from scratch, the placement of each node is completely independent from all the others. Bézier curves also make it easy to animate a drawing line because the curves are rasterized as line segments.

In our algorithm, we first determine the control points of the curlicues and the parameters necessary for determining the spine of the spiral in the shape calculation stage. Then the original and following lines are drawn in the animation stage.

The first version of the Double Meandering Algorithm was developed using the Processing API. It has since been ported to iOS devices in Objective C.

4.1. Structure

We generate each iteration of curlicues using four cubic Bézier curves, labeled $C_1$ though $C_4$ (Figure 3a). Each cubic Bézier curve is modified by four control points, which will be labelled $cp_1$, $cp_2$, $cp_3$ and $cp_4$ represent the endpoints of the line, while $cp_2$ and $cp_3$ are used as tangent vectors that control the trajectory of each end of the curve.

Thus for each iteration we need to assign coordinates to 16 control points, four per each curve. Each curve shares it’s $cp_1$ with the $cp_2$ of the previous curve and it’s $cp_4$ with the $cp_1$ of the next curve, thus the duplicate endpoints that can be treated as a single entity we call $p_i$, for $i \in [1, 4]$ (Figure 3b). The new points, $p_1$ through $p_4$ (and $p_{4n}$, representing the $p_4$ of the next curlicue), are also attached to two tangent control points, i.e. the $cp_3$ of the preceding curve and to $cp_2$ of the next curve. The coordinates of these two control points are always set to be the reverse of one another, thus maintaining $C^1$ continuity between the previous curve and the next. We note that a higher order spline curve (such as a NURBS) may replace the four curves, however here we document our original approach. The spiral is defined by at least 6 iterations of curlicues.

4.2. Shape Calculation

In the first stage of our algorithm we set up the random variables that characterize the control points and spine of the spirals. First, we randomly choose the main direction for the spine of the spiral based on one of three types, illustrated in Figure 4. Each spiral type is based on which curlicue point acts as the “attractor” or shift direction of the growth of each iteration, either $p_2$, $p_3$ or $p_4$.

Next we initialize the variables of our spirals with randomized values within the ranges listed below.

- Seed coordinates of the spine, i.e. centre position of the first curlicue, $C$
- Radius of the curlicue, $r$, a value between 0.5% and 4% of the screen width
- Additional length of spine, between 1% and 2% of the $r \times i$, the current iteration
- Growth of radius during growth iterations, between 60% and 140% of $r$
- Number of growth iterations, between 1 and $g$, for $g \in [3, 5]$
- Attractor tilt shift, between $-797.5\%$ and 12.8% of $r$, depending on spiral type
The seed coordinates for the center of the first curlicue are restricted to values that keep the center of the spiral on the screen. Additionally, for the first spiral, we further restrict its position and size such that the whole spiral fits on screen, taking into account the size and type of spiral. We relax this restraint for subsequent spirals, creating a parameterization such that a large percentage of the spiral will be drawn on screen.

The drawing of a spiral consists of two phases, similar to the process as described by the artist: growth iterations (Opening Stage) and warp iterations (Closing Stage). During the growth iterations (or the opening of the spiral), the radius of each respective curlicue increases according to the initialized radius growth multiplier. A spiral consists of a randomly determined number of growth iterations between 1 and \( g \), for \( g \in [3,5] \). In each growth iteration, we process the Bézier curve control points of the associated curlicue. The control points are initialized to lie in a circle of radius \( r \) around the current curlicue center, \( C \). Then, we alter the position of the control points based on the attractor tilt shift, as shown in Figure 5. Finally, we increment the center of curlicue to the next coordinate on the spine and, if this is a growth iteration, increase the radius.

During the warp iterations, or the closing of the spiral, the radius ceases to grow, but the spine changes trajectory and the shape of the curlicue becomes more and more exaggerated towards the attractor. We found that algorithmically determining the number of warp iterations (or closing of the spirals) was difficult and found through trial and error that six warp iterations, followed by the tail most closely replicated the behavior of the spirals found in Lucy Pullen’s drawings. When we reach the end of the warp iterations, we use one last iteration to make a tail. We achieve this by setting the control points before the attractor like a regular warp iteration. Then, we multiply the coordinates of the attractor by a scale factor of suitable magnitude (default setting is 300%), and set all the control points after it to those same multiplied coordinates. This gives us a ‘tail’ that follows the same trajectory as the final attractor.

4.3. Drawing and Animation
The ultimate goal of the digital Double Meandering Algorithm is to create an animation that conveys the process of the drawing, rather than emphasizing the finished project. To recap, the original line and following line in a finished Double Meandering Line drawing do not come across as independent entities. Instead, they tend to form a single double-lined line entity, perhaps even appearing as the outline of a rope. Yet, the separate natures of the original and following lines is an intrinsic part of the philosophy behind the Double Meandering Line drawing. Hence, our primary objective is to convey the leading nature of the original line and the subordinate aspect of the following line in the animation.

As Bézier curves are traditionally drawn in line segments,
we can easily draw partial curves simply by drawing more or fewer segments. A drawing-in-progress animation can thus be achieved simply by increasing the number of drawn segments from frame to frame. We employ the same principle in reverse to ‘un-draw’ the spiral, allowing for theme-appropriate erasure.

The original line is defined by the shape calculation step discussed in Section 4.2. The following line simply is drawn with an additional Bézier curve chain parallel to the original line. To convey the separate and subordinate nature of the following line, the following line animation lags two full curves behind the first. This gives an orbiting appearance to the animation, as the two drawing lines are directly opposed to each other at all times during the drawing animation.

We must also deal with the illusory depth of the Double Meandering Line drawing. Although our goal is a 2D drawing, we can easily use the benefit of a 3D rendering buffer and we do not need to calculate depth values in a complex manner. Instead we use orthographic projection and a slight offset into the screen for each iteration to automate occlusion. We replicate the overlapping in our algorithm simply by receding each line segment into the background by a single pixel. We also draw a background-coloured strip that accompanies the following line as it is being drawn. This strip lies between the two lines and covers the curlicues that recede further into the background, thus simulating the requisite empty space. We negate the expected perspective distortion on the later iterations of the spiral by using orthographic projection to render our image.

5. Comparison

The Double Meandering Line begins in the first dimension with a point, graduates to the second dimension with a line, and to three dimensions with a second line. As our paper demonstrates, the Double Meandering Algorithm takes this progression one step further towards a four dimensional drawing in time. This brings a whole new range of aesthetic considerations to the table—removal of old spirals, ideal spiral placement and scale, and ideal speed of animation.

Since we design the animation to run into perpetuity, we have to remove the spirals from the screen in a pleasing way in order to showcase new ones. We achieve this by having the lines ‘un-draw’ themselves, erasing themselves from the screen in the same order as they were drawn. Alternate methods could include fading old spirals into the background as new spirals are drawn, or simply flushing the screen after all the spirals have finished drawing.

We are actively investigating what increases or decreases the sense of 3D space in the Double Meandering Algorithm drawings. This is a compositional problem which is solved through the artist’s conscious choices, and is not so easy to quantify. One observer mentions that in order to establish the illusory 3D scope in which the drawings exist, the drawings appear to require at least two spirals with almost perpendicular spines to be on the screen at the same time. It also appears to be necessary to balance the sizes of all three spirals to consistently offer a sense of relative scale. This is one of the many cases where authenticity and randomization appear to be at odds with one another.

The current speed of animation is arbitrarily chosen and depends largely on the speed of the rendering engine. The Processing 3D API in Java takes approximately 2:41 min-
utes to complete the drawing and erasure of three spirals. While the rush of a faster progression is more aesthetically satisfying, the current speed is more accurate to the speed of the original pen-and-paper drawing and showcases the differences between each successive curlicue. Whether or not the ideal speed would be faster than the current one, change of this element requires the animation to be modified to fit another engine.

6. Conclusions

The Double Meandering Algorithm is not ‘canned chance’ but a concatenation of singular aesthetic events. The pen-and-paper drawing is static. The algorithm is diachronic – the singular nature of each spiral becomes apparent in time. Increasing the amount of time one can spend with the operation is worthwhile. The result is what you might call a concatenation of singularity [Rau10], or a chain of distinct events that form a new thought. As aesthetic experiences go, watching the transition from one event to another is satisfying.

Figure 7: An analog Double Meandering Line drawing by Lucy Pullen with a more complex base.

In her later Double Meandering Algorithm drawings, Pullen explores the ‘meandering’ element to a much greater extent, bringing the third dimension into play and sometimes abandoning the spiral structure altogether after the initial loops. Our algorithm is completely unable to keep up with the growth and expanse of her vision, due to the huge discrepancy between the rigidity of the digital Double Meandering Algorithm rules and the free-flowing, periodically reinforced rules of the later analog Double Meandering Line drawings. In the future we will explore the parameter space of our algorithm and look forward to increasing our algorithm’s flexibility to both mimic the same evolutionary steps experienced by Pullen’s more recent drawings and to maybe take some steps that may not be easily explored in hand drawn Meandering Lines.

References


